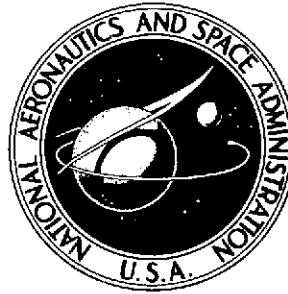


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# EFFECT OF INERTIA PROPERTIES ON ATTITUDE STABILITY OF NONRIGID SPIN-STABILIZED SPACECRAFT

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## CONTENTS

	<i>Page</i>
ABSTRACT . . . . .	i
INTRODUCTION . . . . .	1
DISCUSSION OF ENERGY DISSIPATION PHENOMENA . . . . .	2
CONCLUDING REMARKS . . . . .	13
REFERENCES . . . . .	15

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# EFFECT OF INERTIA PROPERTIES ON ATTITUDE STABILITY OF NONRIGID SPIN-STABILIZED SPACECRAFT

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## INTRODUCTION

This document is intended to clarify and define stability criteria relating to spin-stabilized spacecraft, with particular reference to the criticality of inertia ratios, and the rate of stabilizing to a condition of spin about the axis of maximum inertia. The information given should assist in the critical design review of spacecraft by assessing their susceptibility to stability degradation produced by energy dissipation phenomena.

Two situations are significant for real spacecraft of finite rigidity: The first is a minor coning deviation from stable spin about the principal axis of maximum inertia. This wobble perturbation will damp out asymptotically. Rapid decay of the coning is normally desirable and is promoted by a large excess of spin-axis inertia over the inertias about both the orthogonal lateral axes and by a high rate of energy dissipation from the spacecraft. This coning condition can be initiated by perturbations induced by events that occur during or shortly after orbital insertion of a spacecraft. The result of such perturbations is a spin vector inclined to the spacecraft principal axis of maximum inertia, which is not a stable condition for a dissipative system. The spin axis will precess in a cone of decreasing apex angle with apex at the spacecraft mass center until stable spin about the principal axis of maximum inertia is established. The system angular momentum vector remains invariant and fixed in space during this transient coning decay.

The second case is a minor coning deviation from the condition of spin about the principal axis of minimum inertia. This wobble perturbation will increase with time and "spiral out," eventually degenerating to a flat spin or tumble about the major inertia principal axis. In this case, it is desirable that the inevitable buildup of coning action should be as slow as possible. The preferred inertia distribution is for the spin inertia to be much less than either of the two lateral inertias, and the rate of energy dissipation from the spacecraft should be as low as possible. This situation may be visualized as similar to the first case, but with the time sense reversed. An ideal initial condition with perfect alignment of the spin axis and principal axis of minimum inertia never exists in fact. The spin axis will precess in a cone of increasing apex angle about the invariant, space-fixed angular momentum vector.

The first case may be visualized as similar to the behavior of a rapidly spinning top when hit from the side. A wobble will result, but will damp out to restore the initial condition. The second case is like the behavior of a spinning top in the terminal stage, where it exhibits increasingly severe wobble and finally falls over. This should be understood as a

crude analogy useful for visualizing the motions rather than as a valid mathematical model, since the top is not a free body and is subject to gravity-torques-induced precession.

In both spacecraft situations, energy is dissipated as a result of the spin vector being perturbed within the spacecraft. A free body with initial spin conserves angular momentum and also tends to a condition of minimum kinetic energy, which is steady state-spin about its major principal axis.

A third condition is significant and it should be avoided by design. This is spin about an axis close to the principal axis of intermediate inertia, which can be shown to be inherently unstable even for a nondissipative system. Thomson (Reference 1, page 130) presents an analysis of the instability of this condition. For a dissipative system, large erratic tumbling motions will build up very rapidly, since they are required to retain angular momentum for even minimal energy dissipation.

## **DISCUSSION OF ENERGY DISSIPATION PHENOMENA**

For a given oscillatory stress condition, the energy dissipation rate depends on the compliance and damping characteristics of the spacecraft. These characteristics are difficult to evaluate, either analytically or experimentally. Actual dissipation rates for a spacecraft subjected to free coning motion are usually too low to produce any readily detectable effect under laboratory conditions, and the causative mechanisms may be unknown and/or highly complex. These comments may not apply to devices intended to operate as energy dissipators, such as nutation dampers.

In all cases, energy dissipation is caused by oscillatory angular motions about the mass center of the spacecraft. These motions have components about the spin, pitch, and yaw axes. For coning motion with a half-angle of  $5^\circ$  or less, the motion of any point in the spacecraft is essentially in a plane normal to the radiant from the mass center to that point. Velocities, accelerations, and, therefore, induced forces are also in this plane and their magnitudes are proportional to the distance from the mass center. However, for most energy-dissipation mechanisms the dissipation rate at a point is proportional to the square of its distance from the mass center.

Now, consider how the oscillatory motion causes energy dissipation. If the spacecraft were completely rigid, its motion would be monolithic, with no relative motion between its parts, and its kinetic energy would remain constant. However, deformations will occur in a non-rigid body, causing some kinetic energy to be converted into strain energy and viscous or friction effects. To the extent that deformation is elastic, the energy is recoverable, but with damping there is a net loss of kinetic energy during each cycle. To conserve angular momentum at a lower kinetic energy level, there must be a coning angle change. A flexible body with zero damping, which is as unattainable in practice as a perfectly rigid body, would not dissipate energy. Therefore, both stiffness and damping properties of the structure affect the energy dissipation rate.

## **Dissipative Mechanisms**

Spacecraft are, in general, very nonhomogeneous objects, and much of the total energy dissipation is likely to result from dissipative mechanisms in localized areas. The challenge to the design reviewer is to recognize possible mechanisms and assess their effect on flight behavior.

Some common dissipative mechanisms are as follows:

- Internal friction loss due to cyclic stresses of elastic structure. Some loss is always present, but is usually comparatively small unless there are large deflections involving material with high damping properties. High loss-rates are often associated with large deflections due to resonant response.
- Sliding friction between parts, which can be estimated on the basis of interface loading, friction coefficient, and range of possible motion. This dissipative mechanism may occur in cyclic bending of multiconductor cable bundles, or in appendage hinge points.
- Laminar viscous shear of liquids. This can be an efficient dissipator, depending on viscosity and geometric flow constraints, and has often been used in nutation dampers. Devices using this dissipative mechanism may be tuned for resonance at a particular spin rate.
- Turbulent fluid action. This mechanism can also be efficient and can be tuned for resonant response.
- Cyclic yield of viscoelastic material, such as plastic electrical insulation. Damping can be comparatively high for such materials.
- Impact loss effects caused by repetitive collisions, such as a slug impacting the ends of a closed tube, or shaking of liquid in a closed container violently enough to produce bulk fluid transport with periodic splashing against container walls. Damping is a function of momentum of the free moving parts and restitution coefficients at collision.
- Eddy current losses, due to parts moving in a magnetic field generated within the spacecraft. This has been used in nutation dampers.

## **Kinetic Energy Transfer Induced by External Forces**

The preceding mechanisms are caused only by the gyrodynamic motion of a free body. Although it is common practice, and often justifiable, to consider an orbiting spacecraft as a classic free body, there are many cases where dynamic behavior is affected by environmental influences that apply torques to the spacecraft. Some examples follow below.

- Torques produced by an active control system. Such effects are the result of a specific design intent. A design review should consider the possibility of inadequate

capability and/or malfunction. One Applications Technology Satellite (ATS-5) was an example of a case where the ability of an active control system to override degenerative dissipation was overestimated because a major dissipative mechanism was overlooked—the fluid motion in heat pipes.

- Thermal effects due to solar exposure, with periodic shading. This is probably the major environmental influence which causes behavior to differ from a free-body situation. The significant factor is usually dimensional changes induced by periodic variation of temperature differentials. The result could be degenerative or regenerative depending on spin rate, thermal lag, solar aspect angle, and spacecraft geometry.

Long, slender, flexible appendages with low thermal mass are particularly susceptible. An example of this kind of effect was a Small Standard Satellite (S<sup>3</sup>-A), which developed degenerative coning because of thermally induced boom bending even though it had a favorable inertia ratio. The problem was corrected by a change of solar aspect angle and a change of spin rate. The latter detuned the thermal variation frequency from the natural bending frequency of the booms.

- Interaction with ambient magnetic field. This can be used for attitude control purposes. It can usually be ignored for essentially nonmagnetic spacecraft.
- Gravity gradient effects. Generally these are significant only for very large spacecraft. Such spacecraft often tend to be relatively flexible.
- Solar-wind or light-pressure influence. These phenomena can change spin rate over a long period. The effects could be significant on spacecraft with large areas of exposure.
- Residual atmospheric forces. These are most likely to be detrimental, especially for high-drag configurations with low orbit or at least low perigee.

### Quantitative Evaluation of Energy Dissipation

Quantitative evaluation of the energy dissipation rate can be difficult even for simple systems intended to function as dissipators. For a complex structural system, the task can defy analysis or experimental measurement. However, it is possible to define the dependence of dissipation rate on some system parameters, including the inertia ratio  $K$  between spin and pitch inertias, the spin rate  $\omega$ , the semi-coning angle  $\theta$  and its time derivative  $\dot{\theta}$ , and the rotational kinetic energy of the system  $E$ . The time derivative  $\dot{E}$  is the energy dissipation rate.

Figures 1, 2, and 3 illustrate these functional relationships graphically. On the basis of the referenced literature, the following generalized equations can be inferred:

$$\dot{E} = - \left| C \omega^3 K^n (K - 1) \sin^2 \theta \cos^3 \theta \right| \quad (1)$$

$$\dot{\theta} = \pm \frac{C}{I} \omega^{3-2} K^n \sin \theta \cos^2 \theta \quad (2)$$

where  $\dot{\theta}$  is positive for  $K < 1$  and negative for  $K > 1$ .

Equations (1) and (2) are related by the general relationship

$$\dot{E} = \dot{\theta} [I\omega^2(K-1) \sin \theta \cos \theta] \quad (3)$$

Equation (3) is based on basic energy and momentum considerations and applies to all free-body systems. It is also universally true that the total energy  $\Delta E$  available for spin decay dissipation is

$$\Delta E = \frac{I\omega^2}{2} (1 - K) \quad (4)$$

Interpretation of these equations and of Figures 1, 2, and 3 requires definition of the various symbols:  $I$  is the spin-axis inertia of the system,  $a$  and  $n$  are exponents between 5 and 2, and  $C$  represents an overall energy dissipation rate factor for the specific model. For example, in the analysis by Thomson (Reference 1), it was derived by assuming a homogenous hysteretic damping factor and elastic modulus, integrating the squared stress distribution over the structure, and dividing by the precessional frequency. In this case, hysteresis strain energy loss was assumed to be the energy dissipation mechanism. For the purpose of this discussion, it is assumed that  $C$  represents factors determining  $\dot{E}$ , which are independent of  $K$ ,  $\omega$ , and  $\theta$ .

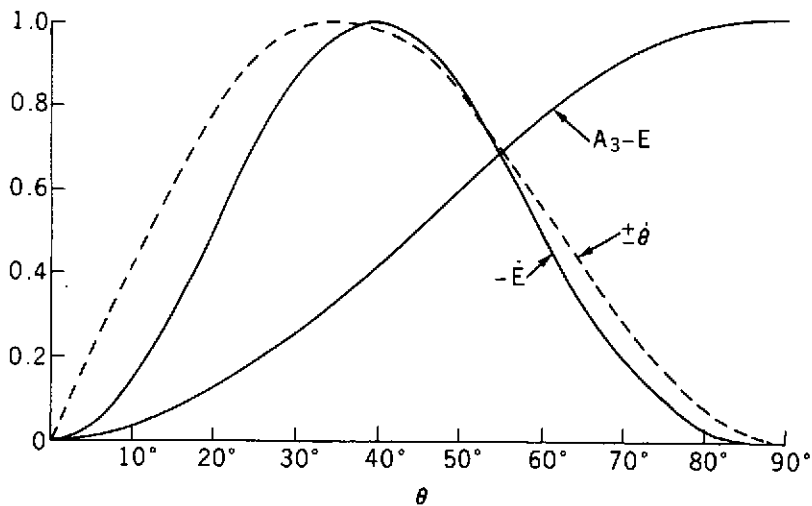


Figure 1. Energy dissipation rate and coning angle rate of change versus coning angle.



It is also assumed that the system has pitch inertia equal to yaw inertia and that the spin-axis principal inertia is  $I$ .  $K$  is the inertia ratio. By definition, for  $K > 1$ ,  $I$  is the system's unique maximum inertia, and for  $K < 1$ ,  $I$  is the unique minimum inertia. By definition,  $\theta$  is the deviation of the axis of  $I$  from the spatially invariant, system-angular-momentum vector. The angular velocity about the axis of  $I$ , when  $\theta = 0$ , is  $\omega$ , whether this is the initial or the final condition. At this point, the total kinetic energy of the system  $E$  is  $I\omega^2/2$ , which is either a maximum or a minimum energy condition, depending on whether  $K$  is less than or greater than 1.0.

The symbol  $\Delta E$  is defined as the difference between a maximum energy condition (spin about the minor principal axis at maximum steady-state rate) and a minimum-energy condition (spin about the major principal axis at minimum steady-state rate).  $\Delta E$  is therefore the total amount of energy available for dissipation during spin decay.

In practice, the equations or curves apply to situations of interest only for small values of  $\theta$ . As  $\theta$  approaches  $90^\circ$ , they define the behavior of a laterally symmetrical body with spin axis in the vicinity of the plane of its two minor or major principal inertias, such as a slender pencil-shaped rod tumbling end over end, for  $K < 1$ , or a coin-like disk spinning about a diameter, for  $K > 1$ . These are real physical conditions, but seldom have much practical significance. The curves and equations define equally well a "cone in to major axis" case ( $K > 1$  and  $\theta$  decreasing), or a "cone out from minor axis" case ( $K < 1$  and  $\theta$  increasing).

Figure 1 shows the dependency of  $E$ ,  $\dot{E}$ , and  $\dot{\theta}$  on  $\theta$ . For this purpose, Equation (2) becomes

$$\dot{\theta} = \pm A_1 \sin \theta \cos^2 \theta \quad (5)$$

where  $A_1$  is a constant arbitrarily chosen to give a maximum value of 1.0, which occurs at  $\theta = 35^\circ$ .  $\dot{\theta}$  is positive if  $K < 1$  and negative if  $K > 1$ .

For  $\dot{E}$ , Equation (1) becomes

$$\dot{E} = -A_2 \sin^2 \theta \cos^3 \theta \quad (6)$$

where  $A_2$  is a constant chosen to give a maximum of 1.0, which occurs at  $\theta = 39^\circ$ . If the system has no energy inputs,  $\dot{E}$  must always be negative.

For  $E$ , Equation (3) can be written as

$$dE = -2\Delta E \sin \theta \cos \theta d\theta \quad (7)$$

$$E = -\Delta E \sin^2 \theta + A_3 \quad (8)$$

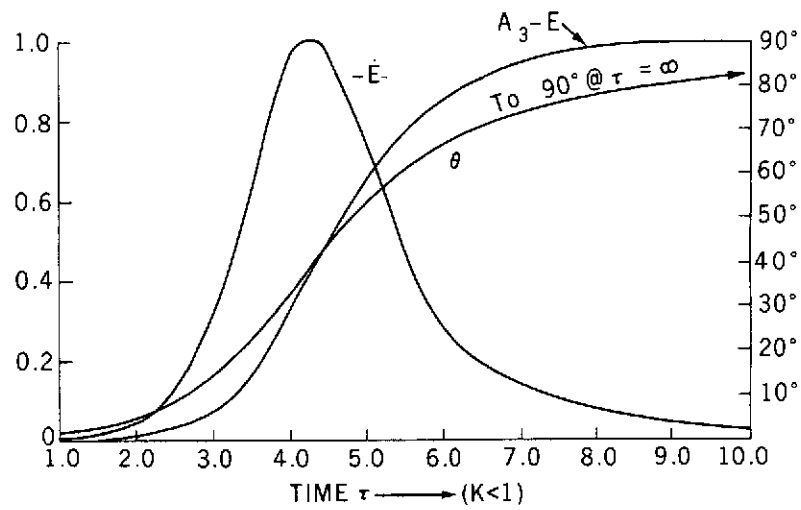


Figure 2. Energy dissipation rate and coning angle versus time.

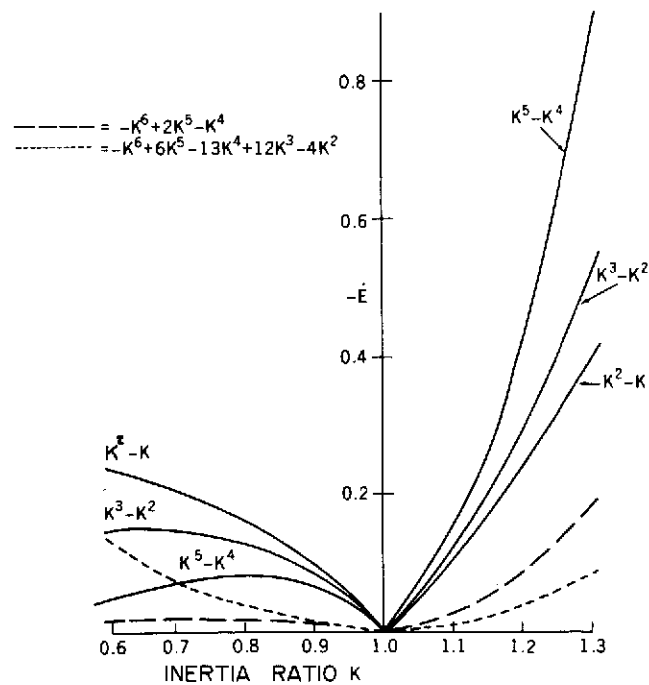


Figure 3. Energy dissipation rate versus inertia ratio.

where  $A_3$  is the energy level of the system with  $\theta = 0$ . In Figure 1,  $E$  versus  $\theta$  is a sinusoidal curve plotted for  $(A_3 - E)$ , with  $\Delta E = 1$ . It may be noted that the energy dissipated in the range  $0 < \theta < x$  is simply  $\Delta E \sin^2 x$ . For  $x = 10^\circ$ , this is about 3 percent of the total available energy. It follows that a small energy loss can imply a large coning angle change.

Figure 2 shows  $\theta$ ,  $A_3 - E$ , and  $\dot{E}$  as time functions. It should be understood that the left to right time sequence applies only for  $K < 1$  cases. If  $K > 1$ , the time sense is reversed and  $\theta$  decreases. However,  $\dot{E}$  is always negative, unless there is an external influence to add energy to the spacecraft, as discussed earlier.

In Figures 3, 4, and 5, the inertia ratio  $K$  is considered a variable, rather than  $\theta$ .  $K$  as defined has the usual meaning of a spacecraft spin/pitch inertia ratio. For this purpose, Equation (1) can be written as

$$\dot{E} = A_4 K^n (K - 1) \quad (9)$$

If constant  $A_4$  is assigned a value of 1.0,  $\dot{E}$  becomes essentially an energy dissipation coefficient dependent on  $K$ , as shown in Figure 3 from  $K = 0.6$  to 1.3 for integral values of  $n$  between 4 and 1. Remembering that  $\dot{E}$  is fundamentally negative, it increases from zero at  $K = 0$  to a maximum, then decreases to zero for  $K = 1.0$ , and finally increases until  $K = 2.0$ . A value of  $K$  greater than 2.0 is not physically possible.

Equation (2) can be written as

$$\dot{\theta} = \pm A_5 K^n \quad (10)$$

With  $A_5$  equal to 1.0,  $\dot{\theta}$  is a dissipation factor dependent on  $K$  as shown by the curves of Figure 4. As  $\dot{\theta}$  is by definition positive for  $K < 1$  and negative for  $K > 1$ , its value at  $K = 1$  is a singularity. From the physical situation, it is logical to deduce  $\dot{\theta}$  to be zero for  $K = 1$ .

The curves of Figure 5, derived as  $\Delta E / \dot{E}$ , are algebraically defined as  $K^{-n}$  and are dimensionally equal to time. They show the comparative time that would elapse for a given portion of the energy dissipation sequence (such as the range between  $\theta = 10^\circ$  and  $\theta = 5^\circ$ ) for different values of  $K$ . This "time factor" is infinite for  $K = 0$  and decreases to a minimum value for the limit case of  $K = 2$ .

Equations (1), (2), and (4) also show the dependency of  $\Delta E$ ,  $\dot{E}$ , and  $\dot{\theta}$  on the basic spin rate  $\omega$ .

At this point, let us examine the application of the foregoing to the critical design review of spacecraft. As an example, assume that spin, pitch, and yaw axis principal inertias are  $I_1$ ,  $I_2$ , and  $I_3$ , with  $I_1 > I_2 > I_3$ , and flight spin rate is  $\omega$ . For some purposes, the inertia ratio  $K$  may be assumed to be  $I_1 / I_2$ , and with this assumption,  $\Delta E$  may be calculated as  $\frac{1}{2} I_1 \omega^2 (1 - I_1 / I_2)$ .

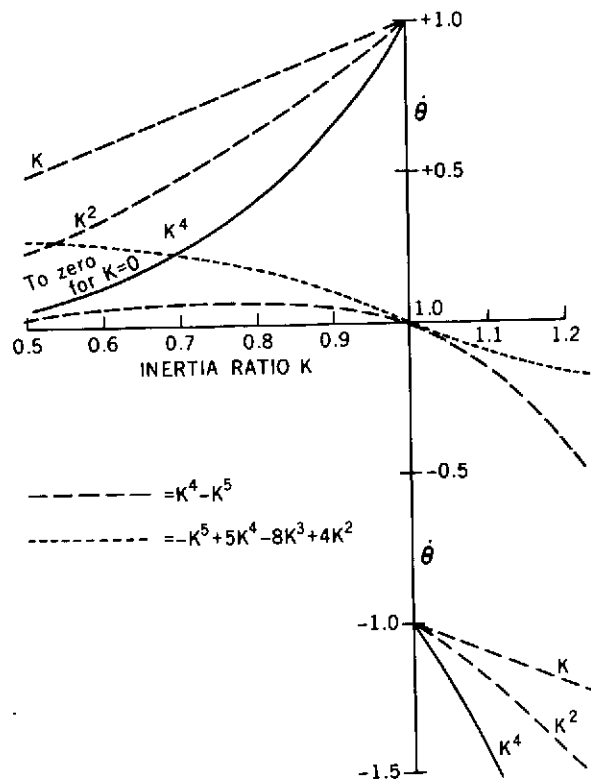


Figure 4. Rate of change of coning angle versus inertia ratio.

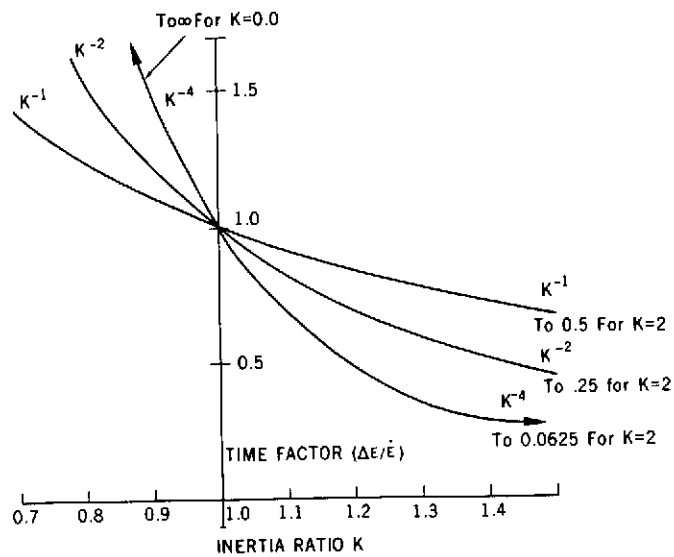


Figure 5. Time factor versus inertia ratio.

If  $0^\circ$  to  $5^\circ$  is considered an acceptable range for coning angle, the energy dissipated over this range is approximately 1 percent of  $\Delta E$  (Equation 8). Also, Figure 5 will show whether the value of  $K$  implies a relatively high or low time factor. The actual rate at which coning angle would diminish requires knowledge of  $\dot{E}$ , which depends on the dissipative mechanisms of the spacecraft. However, if  $0.01 \Delta E$  is divided by an assumed time for  $5^\circ$  coning to reduce to  $1^\circ$  coning, the result will indicate the requisite level of  $\dot{E}$ . This would be an average level and could be assumed to apply to a  $3^\circ$  coning condition for rough estimate purposes. The technique can be applied in all cases; it does not depend on a specific model.

In most cases a significant coning angle change will result from a small loss of energy. For example, assume  $I_1 = 6.8 \text{ kg} \cdot \text{m}^2$  ( $5 \text{ slug} \cdot \text{ft}^2$ ),  $I_2 = 5.4 \text{ kg} \cdot \text{m}^2$  ( $4 \text{ slug} \cdot \text{ft}^2$ ), and  $\omega = 2 \text{ rad/s}$  and the time is 1000 seconds to reduce coning from  $5^\circ$  to  $1^\circ$ . Then  $\Delta E$  is  $3.4 \text{ N} \cdot \text{m}$  ( $2.5 \text{ ft} \cdot \text{lb}$ ), and the implied level of  $\dot{E}$  for  $3^\circ$  coning is  $0.035 \text{ mW}$  ( $2.5 \times 10^{-5} \text{ ft} \cdot \text{lb/s}$ ), or  $1.15 \times 10^{-4} \text{ Btu/h}$ .

A review of representative studies (References 1 through 26) was made to determine what functional relationships have been derived between inertia ratio  $K$  and  $\dot{E}$  and/or  $\dot{\theta}$ . Most of the literature reviewed was concerned with systems including fluids.

The conclusion for the majority of studies reviewed was that  $\dot{E}$  is a function of  $K^2 - K$ , though in two cases (References 2 and 3) it was a function of  $K^3 - K^2$  and the derivation by Thomson for structural dissipation (Reference 1) yielded  $K^5 - K^4$ . The study by Taylor (Reference 25) for a damped spring mass system yields  $-K^6 + 6K^5 - 13K^4 + 12K^3 - 4K^2$  for linear motion parallel to the spin axis, and  $-K^6 + 2K^5 - K^4$  for tangential linear motion.

It appears possible to relate the curves of Figures 3, 4, and 5 to specific types of dissipative mechanism only to a limited degree.

Fourteen of the twenty-five references did not define  $\dot{E}$  or  $\dot{\theta}$  in terms of  $K$ , either explicitly or implicitly. Also, in several cases the inertia parameter used was  $|1 - K|$  rather than  $K$ . It was the algebraic process of expressing  $\dot{E}$  in terms of  $K$ , rather than in terms of  $|1 - K|$ , that led to the seemingly anomalous expressions of Reference 25.

Several studies (References 2, 3, 10, and 24) dealt with a partially or fully liquid-filled toroidal ring as a dissipating mechanism. This mechanism has been used quite extensively as a nutation damper. References 10 and 24 state  $\dot{E}$  as proportional to  $K^2 - K$ , and References 2 and 3 indicate  $K^3 - K^2$ .

For all other systems involving fluids where  $\dot{E}$  was stated in terms of  $K$ , the function was  $K^2 - K$ .

The specific structural model studied by Thomson (Reference 1) led to  $\dot{E}$  being proportional to  $K^5 - K^4$ . It does not seem reasonable to apply this expression to anything other than the specific model geometry (two rigid disks connected by a dissipative flexible tube). The result for Reference 25 is the sole exception to the tentative generality that  $\dot{E}$  is proportional to  $(K^n - K^{n-1})$ . There is some indication from the preceding observations that the

exponent  $n$  is higher for structural than for fluid mechanisms, but even this is speculating a generality from very limited evidence.

In the early stages of this study, it was hoped to reveal some unifying generalities linking  $\dot{E}$  and  $K$  for all systems, or for classified groupings. This was not accomplished; nor is it clear whether future progress along this line is likely or whether future effort would be rewarding.

It can be stated that  $\dot{E}$  is more dependent on  $K$  for some systems than for others and that for real spacecraft with complex nonhomogenous structural geometry, the dependency would be algebraically complex.

### Dual-Spin Systems

There is a special class of spin-stabilized spacecraft for which energy dissipation rates and inertia ratios are important. This is the "dual-spin" or "gyrostat" configuration, where the usual arrangement is for a despun part of the spacecraft to be essentially stationary, while the rest of the spacecraft is a spinning rotor. For a dual-spin system, the inertia ratio  $K$  is defined as  $I_s/I_p$ , where  $I_s$  is the spin inertia of the rotor and  $I_p$  is the pitch inertia of the entire spacecraft, which is assumed to be laterally symmetrical. If  $K > 1$ , the spacecraft is stable. It is also stable for  $K < 1$ , providing that  $|\dot{E}_D| > |\dot{E}_R| K/K - 1|$ , where  $\dot{E}_D$  and  $\dot{E}_R$  are rates of energy dissipation from the despun unit and rotor, respectively. For this type of spacecraft, it is common to ensure that  $\dot{E}_D$  exceeds  $\dot{E}_R$  by a large margin by installing a nutation damper with high energy-dissipation capability on the despun portion and by making the rotor comparatively rigid.

Figure 6 shows  $K$  plotted against  $\dot{E}_D/\dot{E}_R$  for dual-spin cases and shows the region of instability. The essential point is that as  $K$  approaches 1.0, stability can only exist if  $\dot{E}_D$  is very much greater than  $\dot{E}_R$ . Also, a case where  $K$  is believed greater than but very close to 1.0 might be unstable because the accuracy of measuring or calculating inertias is such that  $K$  might actually be slightly less than 1.0.

### Lateral-Inertia Asymmetry

The significance of unequal pitch and yaw inertias will now be discussed. Most analytical studies assume laterally symmetrical inertia distribution, with pitch and yaw inertias equal. This case is easier to solve than the more general case with three different principal inertias, where definition of dynamic motions requires the use of elliptic function integrals. For most purposes, it is convenient, customary, and conservative to treat a case with three unequal principal inertias as if pitch and yaw inertias were equal. The inertia ratio  $K$  is considered to be the ratio between the spin-axis inertia and the intermediate principal inertia.

Many spacecraft are essentially laterally symmetrical, but some are not, particularly configurations having two long, diametrically opposed, experiment appendages—which tend to have spin inertia only slightly greater than intermediate pitch inertia, with yaw inertia much smaller. In some cases, inertia booms have been added along the pitch axis to increase  $K$  and decrease asymmetry.

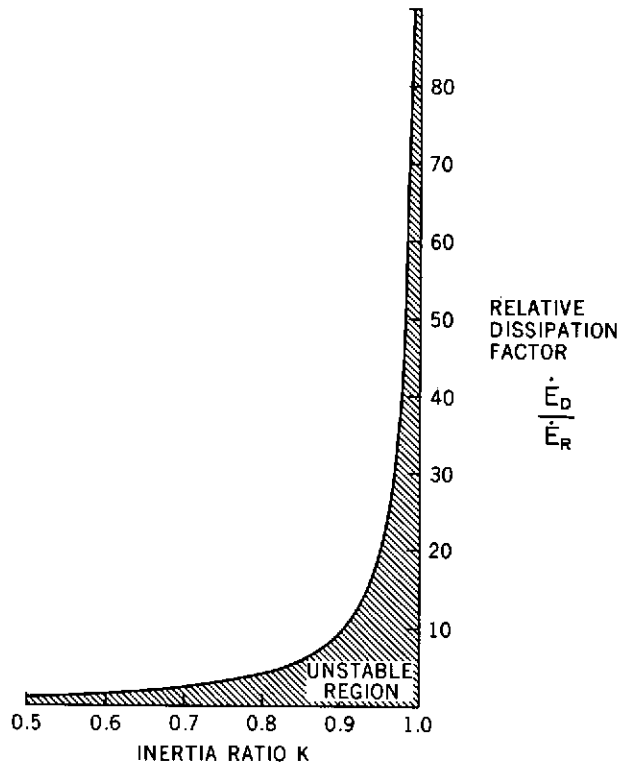


Figure 6. Relative dissipation factor versus inertia ratio for dual-spin systems.

For gyrodynamic motion with small  $\theta$ , the assumption of  $K = I_1/I_2$  in this case is essentially valid. For dynamic-balancing tolerance definition, it would be more logical, though less conservative, to apply  $K = I_1/I_2$  to the unbalance tolerance component in the spin axis/pitch axis plane and  $K = I_1/I_3$  to the unbalance tolerance in the spin axis/yaw axis plane. This would allow more unbalance due to the booms for the "two-long-boom" configurations.

For the influence of lateral asymmetry on energy dissipative characteristics, it would be conservative to assume  $K$  equal to  $I_1/I_2$  with reference to the curves of Figures 3, 4, and 5. Taylor (References 2 and 8) discusses the question of preferential azimuth orientation of nutation damping devices in laterally asymmetrical configurations. While there are preferential orientations for the specific systems discussed, it is not clear whether or to what extent all dissipative phenomena may be influenced by location relative to pitch and yaw axes of asymmetrical spacecraft. It is not possible to infer generalized conclusions from these specific cases. In the viscous fluid ring system of Reference 2, the relative advantage of "best versus worst" orientation is defined in terms of inertia properties, and would not exceed a factor of 5 for any probable degree of asymmetry.

In effect, it is suggested that lateral asymmetry is not likely to significantly affect most dissipative phenomena, though further investigation in this area might be useful.

The preceding comments apply to conditions where the spin vector remains in the near vicinity of the principal axis of either maximum or minimum inertia. Any other condition, including spin in the vicinity of the principal axis of intermediate inertia, is inherently undesirable and the effect of lateral asymmetry would seldom have practical significance.

## CONCLUDING REMARKS

From the viewpoint of a critical design review, the initial problem lies in evaluating the significance of known or previously unsuspected mechanisms for energy dissipation. It is then necessary to consider the analytical or experimental capability for evaluating such dissipative mechanisms. Specific solutions to identified systems will continue to require individual analysis.

In addition to demonstrating the well known fact that inertia ratios in the immediate vicinity of 1.0 should be avoided, this study has provided information to evaluate the relative difference in rate of coning buildup or decay as a function of the inertia ratio  $K$  for specific energy dissipation mechanisms.

The further away  $K$  is from 1.0, the better, so far as the influence of energy dissipation on spin stability is concerned. However, as long as  $K$  is  $>1.05$  or  $<0.8$ , the specific nature of dissipative mechanisms is likely to be more significant than inertia ratio.

There are three other reasons for avoiding inertia ratios close to unity: First, a tolerance margin is necessary to ensure that the system actually is on the proper side of 1.0 after allowing for measurement and/or computational inaccuracies. The second reason is that a system with  $K$  very close to 1.0 has virtually no gyroscopic stiffness, and gyroscopic action is the basic reason for using spin for spacecraft stabilization. The  $\Delta E$  resulting from  $K \neq 1.0$  is essentially an energy source available to resist attitude change. With  $K = 1.0$ , there is no resistance to change of attitude due to external torques.

The third reason has to do with dynamic balancing accuracy necessary to adequately align the principal axis with the geometric axis about which stable spin is desired. The dynamic unbalance tolerance  $D$  is customarily defined by:

$$D = \alpha I_p (K - 1) \quad (11)$$

where  $\alpha$  is the small allowable angular deviation of the principal axis from the geometric axis in radians,  $I_p$  is the pitch inertia (with lateral symmetry assumed), and  $K$  is the inertia ratio  $I_s/I_p$ . For a laterally asymmetrical case,  $K$  is customarily defined as the ratio between the spin axis inertia and the intermediate principal inertia, and  $I_p$  is defined as the intermediate principal axis inertia.  $D$  and  $I_p$  must be expressed in the same units. It is evident



that as  $K$  approaches 1.0,  $D$  becomes unacceptably small. For spin about the intermediate principal inertia axis,  $K$  becomes equal to 1.0 by definition, and this is an inherently unstable condition which must be avoided.

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